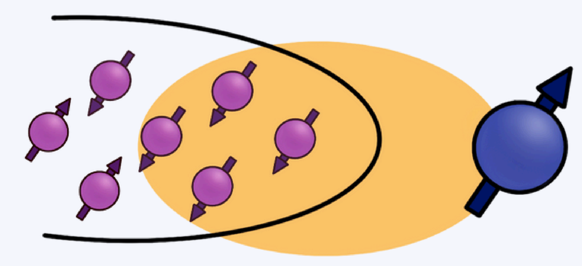
MULTIPARAMETER CRITICAL QUANTUM METROLOGY WITH IMPURITY PROBES

George Mihailescu, Abolfazl Bayat, Steve Campbell, Andrew Mitchell

[1] SINGLE IMPURITY THERMOMETRY

MODEL



PAPER LINK TO [1]

$$\hat{H} = \hat{H}_{bath} + J^z \hat{S}^z_I \hat{S}^z_0 + \tfrac{1}{2} J^{\perp}\left(\hat{S}^+_I \hat{S}^-_0 + \hat{S}^-_I \hat{S}^+_0\right) + B_I \hat{S}^z_I + B_0 \hat{S}^z_0$$

FIGURES OF MERIT

Temperature sensitivity related to probe magnetization $\langle \hat{S}^z_I \rangle$ [1],

$$\mathcal{H}(T) = \frac{\left|\partial_T \langle \hat{S}^z_I \rangle\right|^2}{\frac{1}{4} - \langle \hat{S}^z_I \rangle^2}$$

and characterised by the quantum signal to noise ratio [2],

$$\mathcal{Q} = T\sqrt{\mathcal{H}(T)}$$

impurity-bath entanglement quantified using negativity [3],

$$\mathcal{N}_{I;bath} = \frac{1}{2}\sqrt{1 - 4\langle \hat{S}^z_I \rangle^2} \qquad : T \ll T_K$$

RESULTS

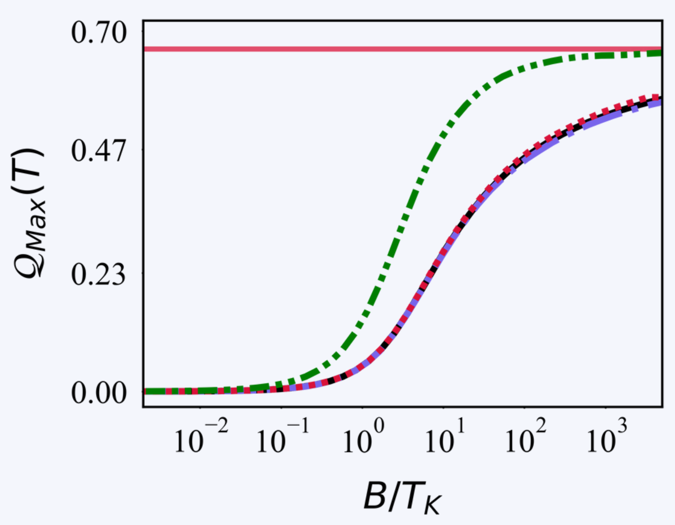


Fig.1. Universal scaling of the Kondo impurity probe for **flat-band**, Gaussian, linear chain, and twisted bi-layer graphene DoS. Ising impurity (free spin) probe response shown in solid red line.

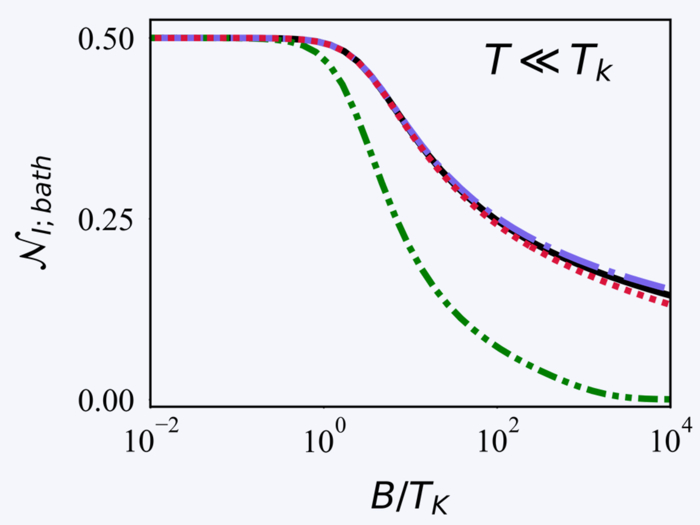
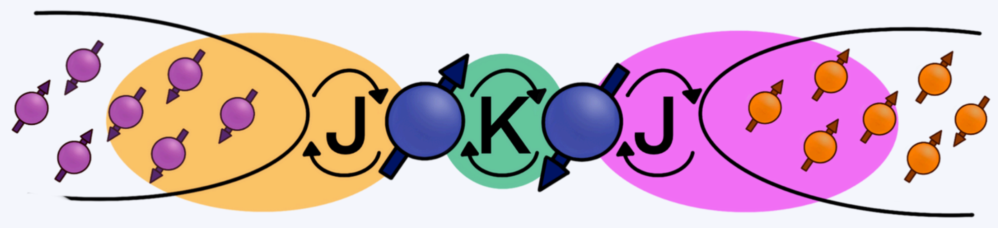


Fig.2. Entanglement negativity decay of the Kondo impurity probe as the applied field strength, B, is increased. Shown for **flat-band**, Gaussian, linear chain, and twisted bi-layer graphene DoS.

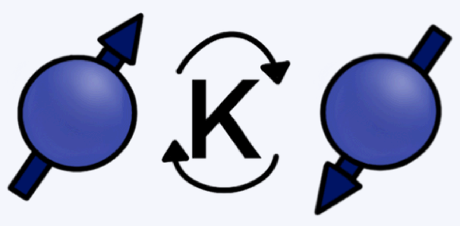
[2] METROLOGY IN A QUANTUM CRITICAL SYSTEM

MODELS



2 Impurity Kondo model

$$\hat{H} = \hat{H}_E + \hat{H}_{\text{probe}} + J\left(\hat{\mathbf{S}}_{IL}\cdot\hat{\mathbf{s}}_{EL} + \hat{\mathbf{S}}_{IR}\cdot\hat{\mathbf{s}}_{ER}\right)$$



Large K-Limit

$$\hat{H}_{KL} = K\,\hat{\mathbf{S}}_{IL}\cdot\hat{\mathbf{S}}_{IR}$$

FIGURES OF MERIT

Parameter sensitivity related to spin-fraction,

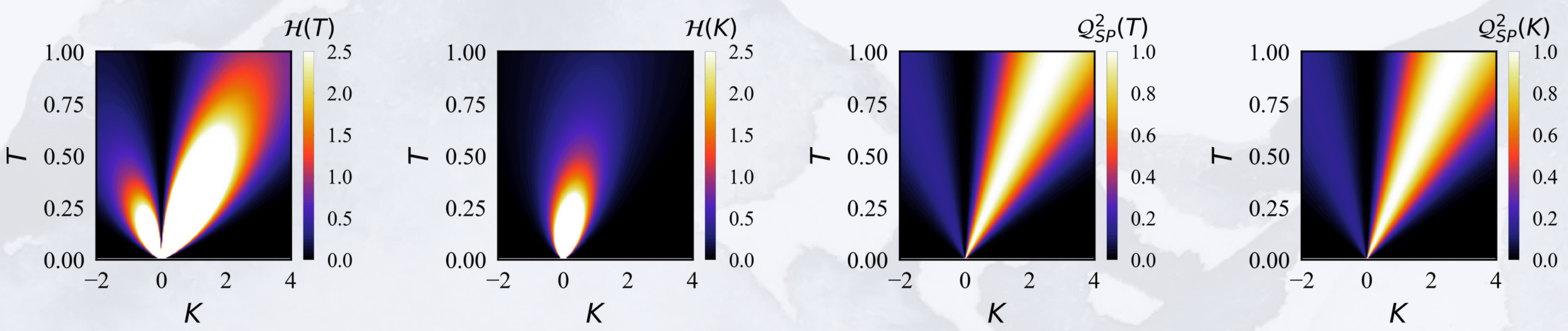
$$\mathcal{H}(\lambda) = \frac{|\partial_\lambda C|^2}{\left(\frac{1}{4} - C\right)\times\left(\frac{3}{4} + C\right)} \qquad \mathcal{Q}_{SP}(\lambda) \equiv \lambda^2 \mathcal{H}(\lambda)$$

where the spin-fraction is defined as,

$$C = \langle \hat{\mathbf{S}}_{IL}\cdot\hat{\mathbf{S}}_{IR}\rangle \equiv \mathrm{Tr}\left\{\left(\hat{\mathbf{S}}_{IL}\cdot\hat{\mathbf{S}}_{IR}\right)\hat{\varrho}_{\text{probe}}\right\}$$

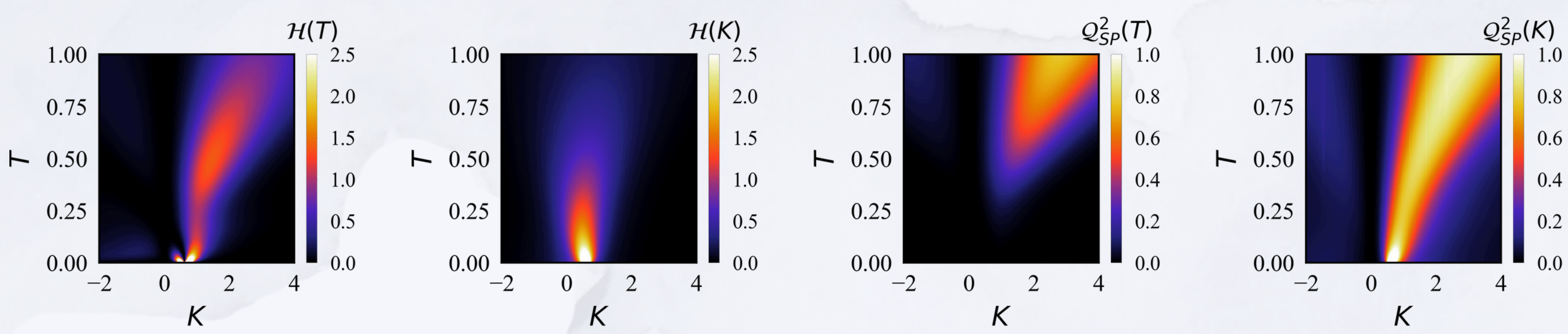
RESULTS: SINGLE PARAMETER ESTIMATION

Large K-Limit



Single-parameter estimation of temperature (T) and impurity-impurity coupling (K) in the large K-limit of two exchange coupled spins extracted from analytical solution.

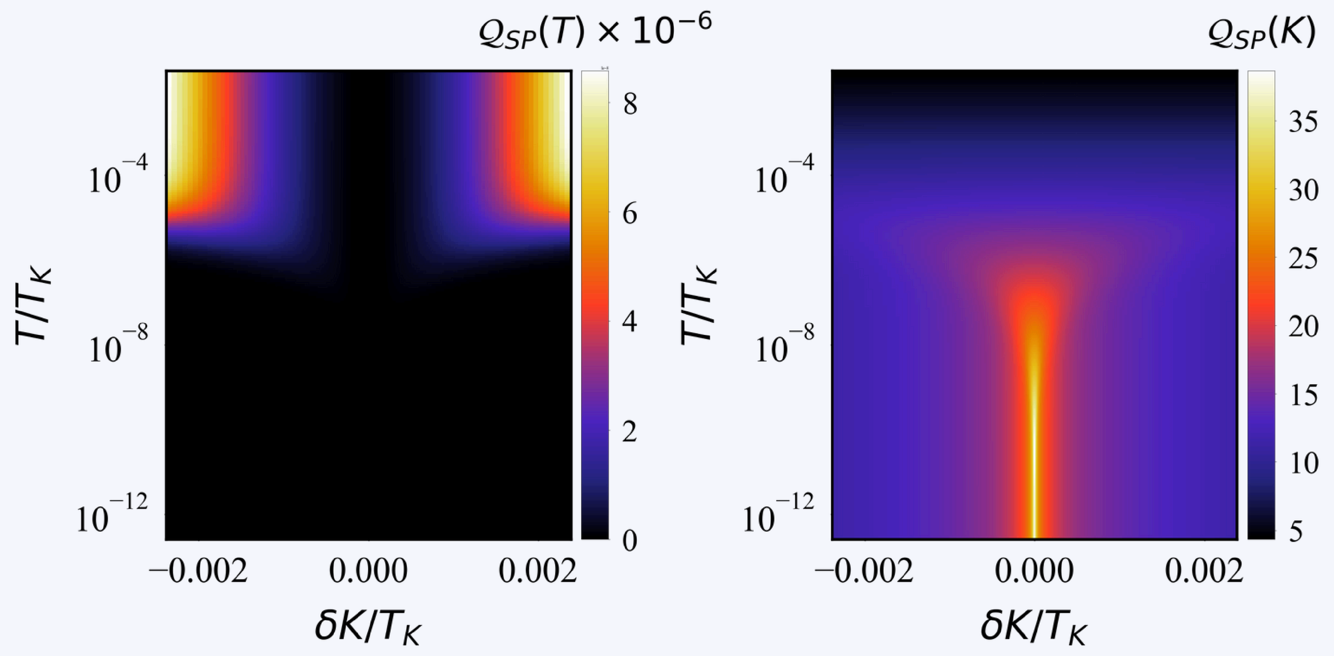
Full 2 Impurity Kondo Model



Single-parameter estimation of temperature (T) and impurity-impurity coupling (K) in the full 2 impurity Kondo model extracted numerically from the Numerical Renormalization Group.

[3] CRITICAL METROLOGY

EXACT RESULTS AROUND CRITICAL POINT



Employing a known analytic solution for the thermodynamic entropy of the model [4], and noting that both the spin fraction C, and the entropy S, have the exact identities,

$$C = \partial_K \mathcal{F}$$

and

$$S = -\partial_T \mathcal{F}$$

allows us to therefore (at thermal equilibrium) apply a Maxwell relation to connect the entropy to the spin fraction,

$$\partial_T \partial_K \mathcal{F} = \partial_K \partial_T \mathcal{F}$$

and solving for suitable derivatives therein allows us to find exact analytic expressions for the relevant QSNRs [5],

$$\mathcal{Q}_{SP}(T) = \frac{4c^2\,\delta K^2\left[T\,T_K - c\,\delta K^2\,\psi'(\Phi)\right]^2}{T^2\,T_K^4\left(\frac{1}{4} - C\right)\times\left(\frac{3}{4} + C\right)} \qquad \mathcal{Q}_{SP}(K) = \frac{4c^2 K^2\left[T\,T_K\left(\log T + \psi(\Phi)\right) + 2c\,\delta K^2\,\psi'(\Phi)\right]^2}{T^2\,T_K^4\left(\frac{1}{4} - C\right)\times\left(\frac{3}{4} + C\right)}$$

[4] MULTIPARAMETER QUANTUM SIGNAL TO NOISE

Question: Typical metrological schemes assume all parameters, except the one being inferred, are known with certainty. How do we incorporate uncertainty from other parameters?

For the multiparameter setting we consider a generalization of the single parameter signal to noise ratio $\mathcal{Q}_{SP}(\lambda) \equiv \lambda^2 \mathcal{H}(\lambda)$,

$$\mathcal{Q}_{MP}(\lambda_i, \lambda_j) \equiv \frac{|\lambda_i \lambda_j|}{[\mathcal{H}^{-1}]_{\lambda_i,\lambda_j}} \geq \frac{|\lambda_i \lambda_j|}{\mathrm{Cov}(\lambda_i, \lambda_j)}$$

where the bounds follow from element-wise manipulation of the CRB. Considering explicitly the estimation of two arbitrary parameters the inverse quantum Fisher information matrix (QFIM) reads,

$$\mathcal{H}^{-1} = \frac{1}{\det(\mathcal{H})}\begin{pmatrix} \mathcal{H}_{\lambda_B,\lambda_B} & -\mathcal{H}_{\lambda_A,\lambda_B} \\ -\mathcal{H}_{\lambda_A,\lambda_B} & \mathcal{H}_{\lambda_A,\lambda_A}\end{pmatrix}$$

where $\det(\mathcal{H}) = \mathcal{H}_{\lambda_A,\lambda_A}\mathcal{H}_{\lambda_B,\lambda_B} - \mathcal{H}^2_{\lambda_A,\lambda_B}$. Thus we find,

$$\mathcal{Q}_{MP}(\lambda_A, \lambda_A) = \lambda_A^2 \frac{\det(\mathcal{H})}{\mathcal{H}_{\lambda_B,\lambda_B}} = \mathcal{Q}_{SP}(\lambda_A) - \frac{(\lambda_A \lambda_B \mathcal{H}_{\lambda_A,\lambda_B})^2}{\mathcal{Q}_{SP}(\lambda_B)}$$

and similarly for $\mathcal{Q}_{MP}(\lambda_B, \lambda_B)$. For the off-diagonal terms,

$$\mathcal{Q}_{MP}(\lambda_A, \lambda_B) = -|\lambda_A \times \lambda_B| \frac{\det(\mathcal{H})}{\mathcal{H}_{\lambda_A,\lambda_B}}$$

For our setup, the multiparameter estimation of temperature and coupling is characterized by a singular QFIM, which is captured in the above expression. By adding a known control field, we recover multiparameter sensitivity:

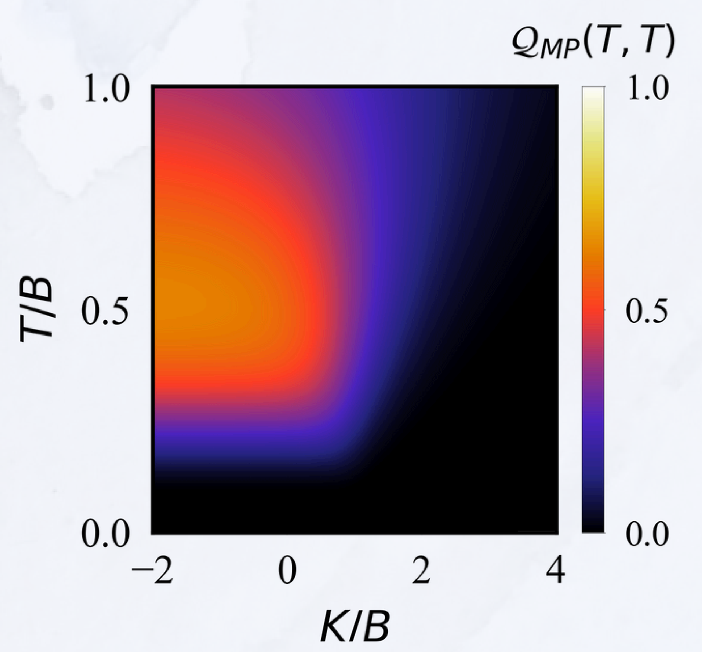


Fig.1. Multiparameter QSNR for thermometry in the presence of a known control field B.

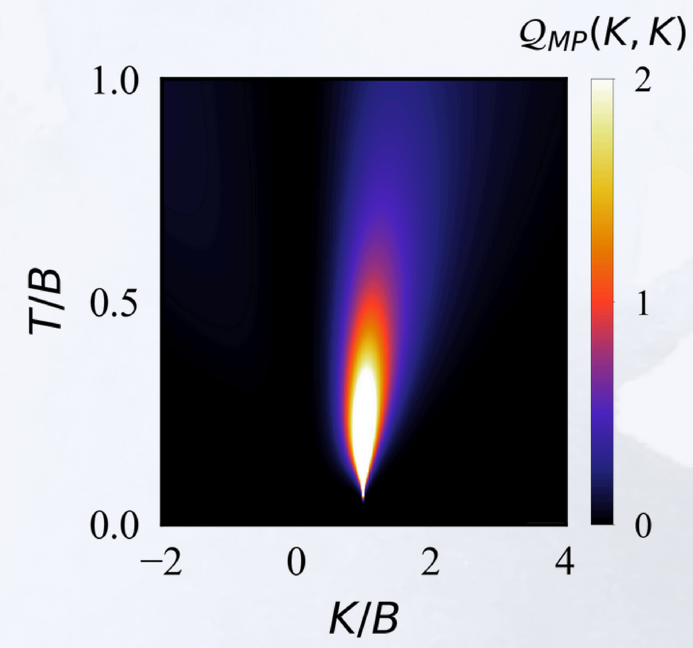


Fig.2. Multiparameter QSNR for the estimation of the coupling K in the presence of a known control field B.

PAPER LINK TO [5]

REFERENCES

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[4] A.K. Mitchell and E. Sela, Phys. Rev. B 85, 235127 (2012)
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Contact: george.mihailescu@ucdconnect.ie