THERMOMETRY OF STRONGLY CORRELATED QUANTUM ENVIRONMENTS

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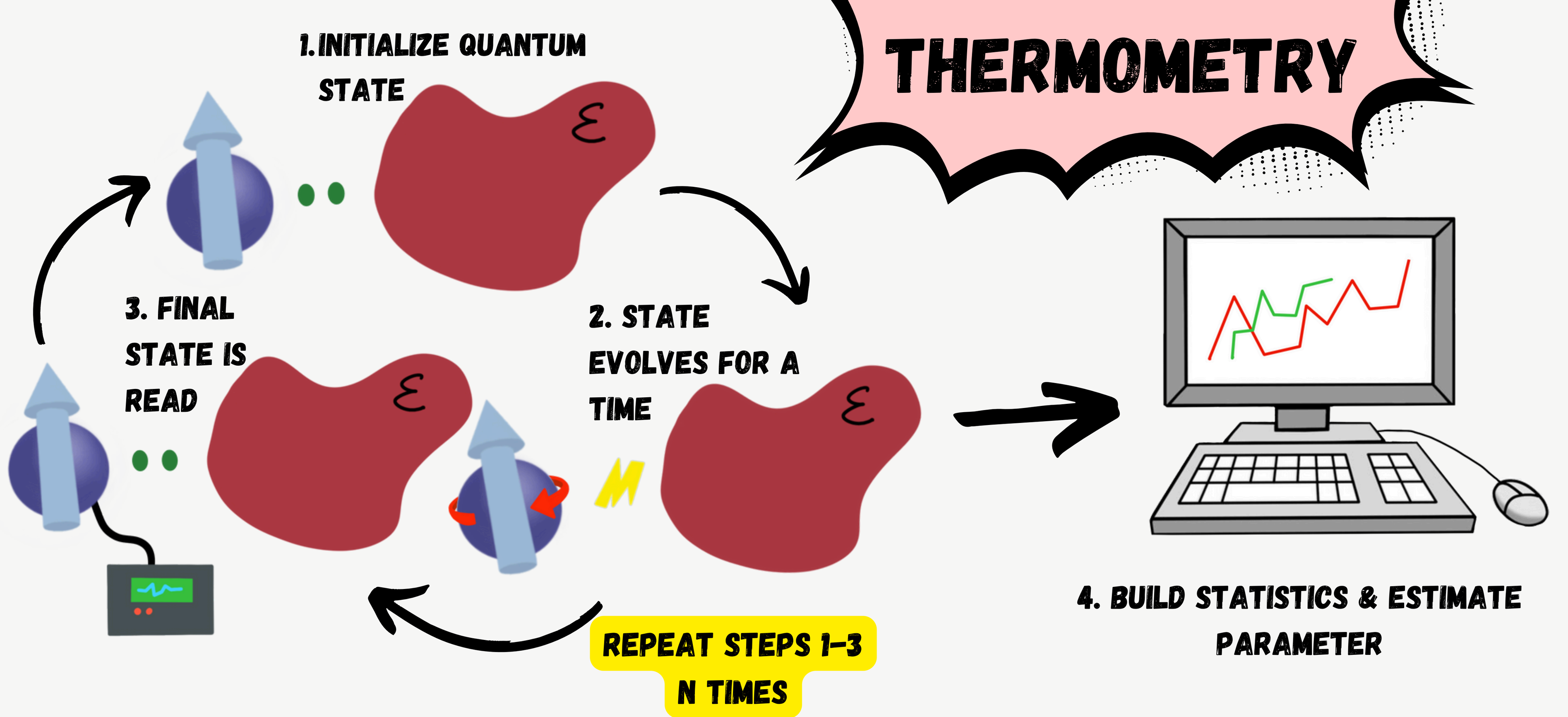
Abstract

We study quantum impurity models as a platform for quantum thermometry. In particular we study how strongly correlated many-body phenomena and the energy structure of the bath influence the impurity probe temperature sensitivity. We find that probe-environment coupling mediated through an Ising interaction results in temperature sensitivity that is invariant to environment details and analogous to a free spin at thermal equilibrium. In contrast, coupling via spin-flip terms which manifests the Kondo effect yields sensitivity that is highly dependant on the environment whose temperature we wish to estimate.

QUANTUM THERMOMETRY



For the estimation of temperature using a probe at thermal equilibrium with some environment, the probe sensitivity is given by the thermal Fisher information [1],

$$\mathcal{H}(T) = \sum_i \frac{|\partial_T \rho_i|^2}{\rho_i} = \frac{\langle \hat{H}^2 \rangle - \langle \hat{H} \rangle^2}{T^4}$$

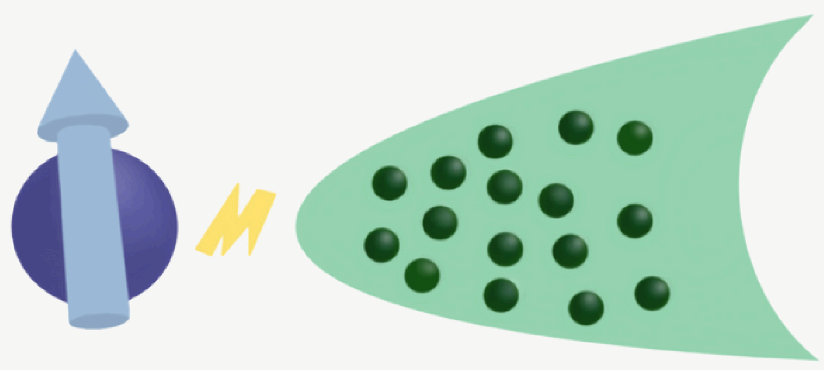
The thermal Fisher information can be expressed in terms of the impurity magnetization,

$$\mathcal{H}(T) = \sum_i \frac{|\partial_T p_i|^2}{p_i} \equiv \sum_{i=\pm} \frac{i \left|\partial_T \langle \hat{S}_I^z \rangle\right|^2}{\frac{1}{2} + i \langle \hat{S}_I^z \rangle}$$

We use the quantum signal to noise ratio to characterise a probes sensitivity to temperature,

$$\mathcal{Q}(T) = T\sqrt{\mathcal{H}(T)}$$

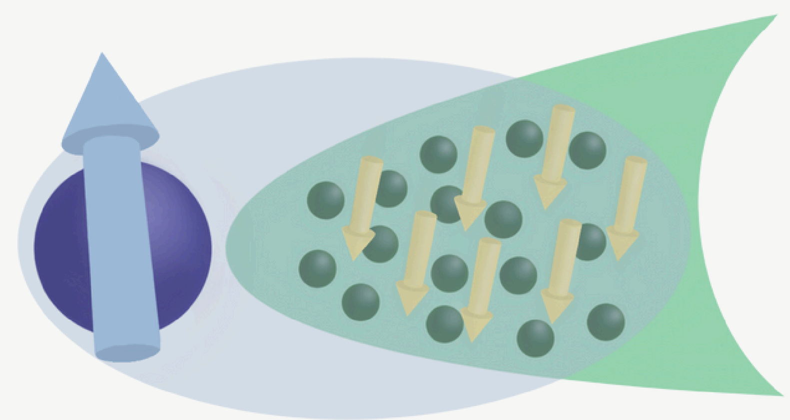
We explore two modalities of impurity-environment coupling [2],



The Ising Impurity model has Hamiltonian,

$$\hat{H}_I = \hat{H}_{lead} + J^z \hat{S}_I^z \hat{S}_B^z + B\left(\hat{S}_B^z + \hat{S}_I^z\right)$$

with Ising interaction J_z^z magnetic field applied to bath and impurity B, and lead structure $\hat{H}_{lead} = \sum_{k,\sigma} \epsilon_k \hat{c}^\dagger_{k\sigma} \hat{c}_{k\sigma}$.



The Kondo impurity model has Hamiltonian,

$$\hat{H}_K = \hat{H}_{lead} + J^z \hat{S}_I^z \hat{S}_B^z + J^\perp \chi + B\left(\hat{S}_B^z + \hat{S}_I^z\right)$$

with $\chi = \hat{S}_I^+ \hat{S}_B^- + \hat{S}_I^- \hat{S}_B^+$ describing spin-flip terms that induce strong impurity-bath correlations at low temperature, resulting in the formation of a many-body singlet state.

ISING IMPURITY

One finds that the free spin model [3] and the Ising impurity model are identical and that probe sensitivity is invariant to the environment whose temperature we are probing.

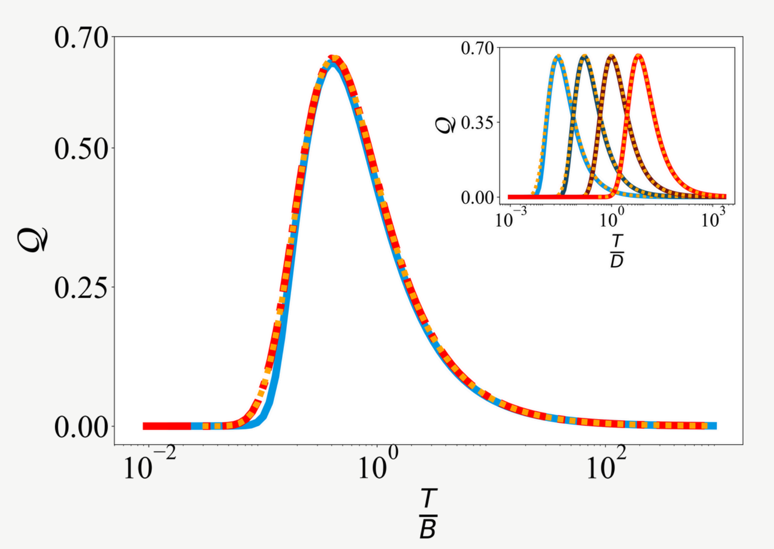


Fig. 1: Ising impurity and free spin temperature sensitivity for weak and strong field strengths.

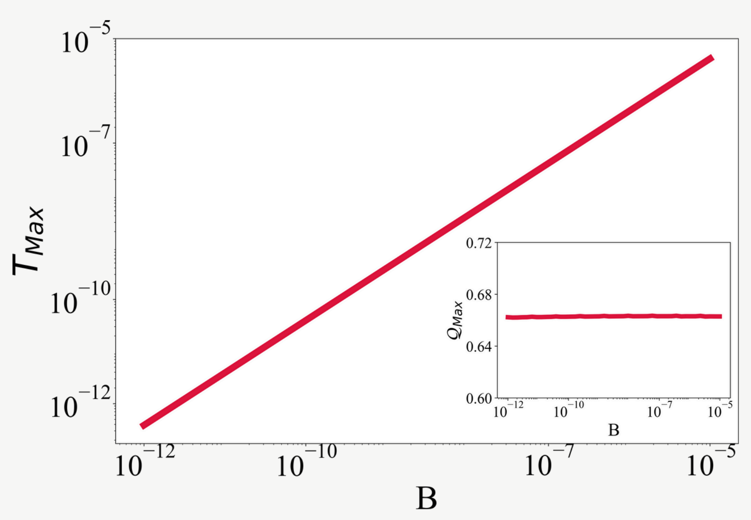


Fig. 2: Temperature at which the Ising impurity sensitivity peaks. ***Inset:*** *Maximum temperature sensitivity of Ising impurity.*

Using a mean field theoretic approach (MFT), one can make the connection between the Ising impurity model and the free spin model more rigorous. The probe temperature sensitivity is related to its magnetization, which from MFT reads,

$$\underbrace{\langle \hat{S}_I^z \rangle = \frac{1}{2}\tanh\left(\frac{B + J^z \langle \hat{S}_0^z \rangle}{2T}\right)}_{\text{ISING IMPURITY MAGNETIZATION}} \overset{\text{SAME AS}}{\approx} \underbrace{\langle \hat{S}_I^z \rangle = \frac{1}{2}\tanh\left(\frac{B'}{2T}\right)}_{\text{FREE SPIN MAGNETIZATION}} \quad \text{with} \quad \overbrace{B' = B + J^z \langle \hat{S}_B^z \rangle}^{\text{RENORMALIZED FIELD}}$$

KONDO IMPURITY

The Kondo model is a strongly correlated many-body problem which is non-perturbative and cannot be approached using a mean field theory. Instead the model is solved using the Numerical Renormalization Group approach [4] to obtain the magnetization exactly.

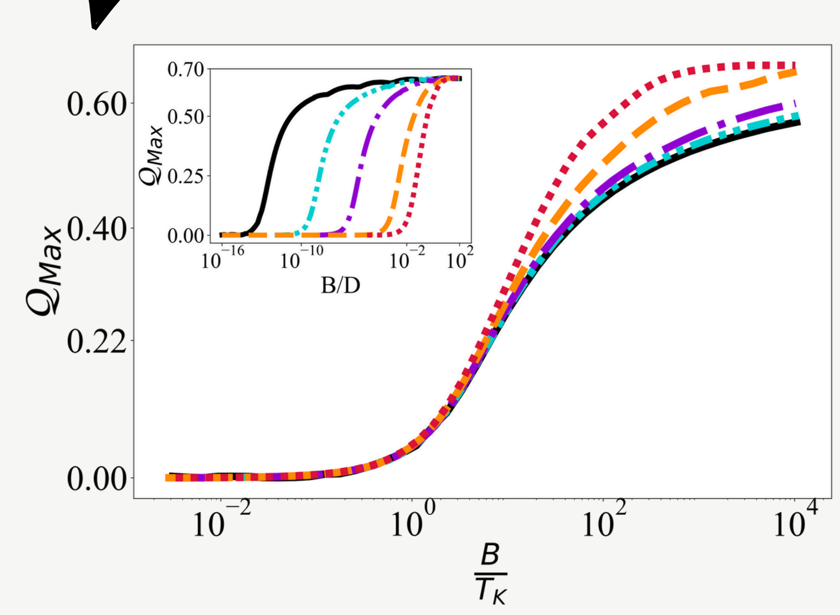


Fig. 3: Maximum Kondo impurity temperature sensitivity for $J^\perp$ = {0.07, 0.1, 0.15, 0.3, 0.5} with a metallic DOS.

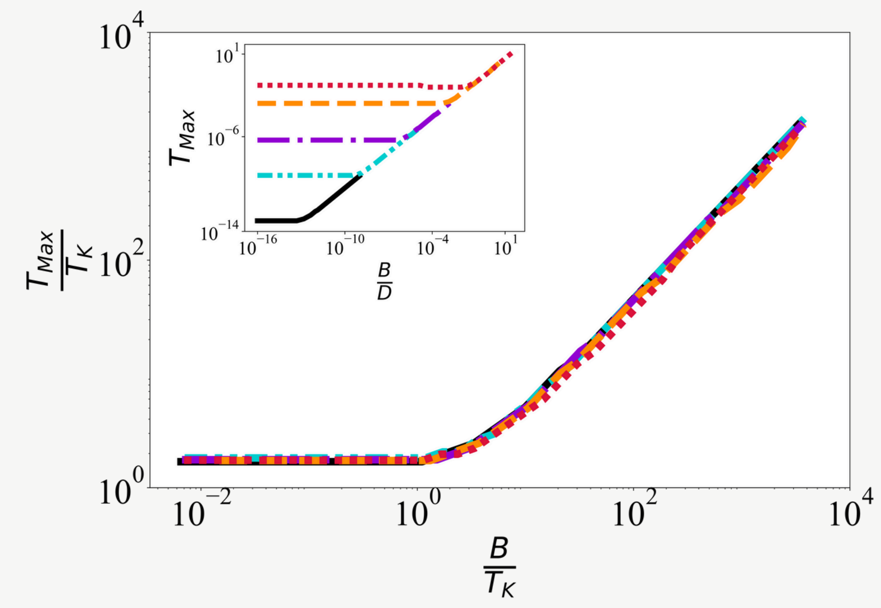


Fig. 4: Temperature of peak sensitivity for Kondo impurity probe for $J^\perp$ = {0.07, 0.1, 0.15, 0.3, 0.5} with a metallic DOS.

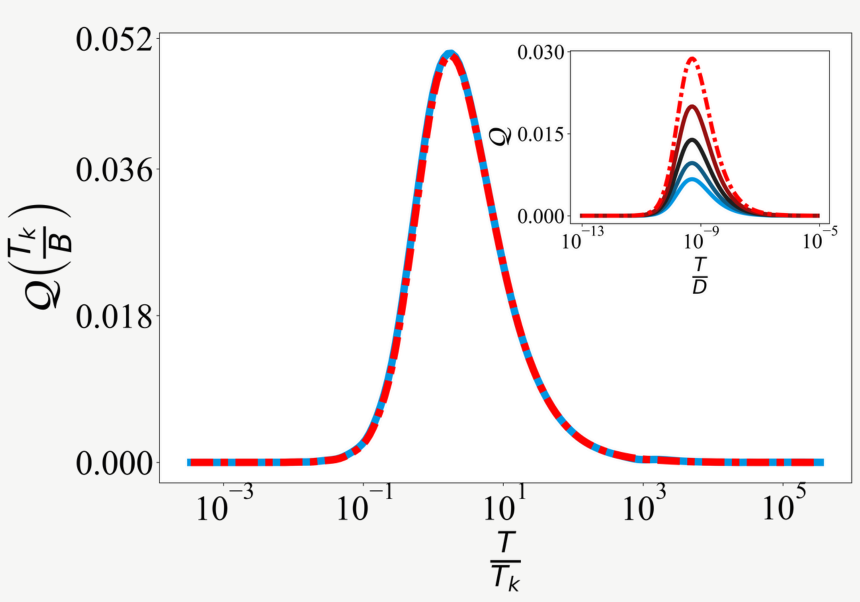


Fig. 5: Kondo impurity probe sensitivity rescaled by the ratio of applied field and Kondo temperature for B << Tk at weak and strong field B. ***Inset:*** *Sensitivity as a function of ratio of temperature to bandwidth.*

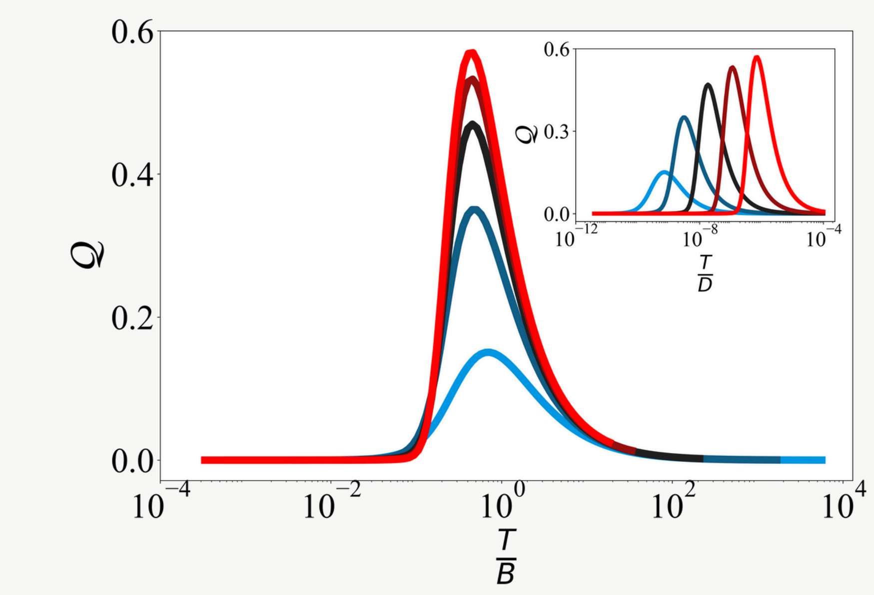


Fig. 6: Kondo impurity probe sensitivity for B > Tk at weak and strong field B. ***Inset:*** *Sensitivity as a function of ratio of temperature to bandwidth.*

DIFFERENT DOS

The Kondo impurity model has a sensitivity profile that is characteristic of the environment that is being probed.

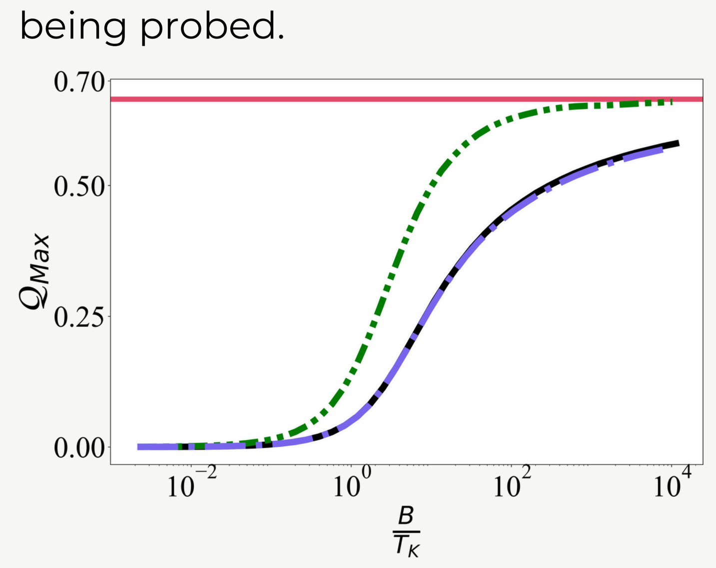


Fig. 7: Max impurity temperature sensitivity in a metallic and twisted bilayer graphene environment. Guide to eye: Max temperature sensitivity for a two-level probe.

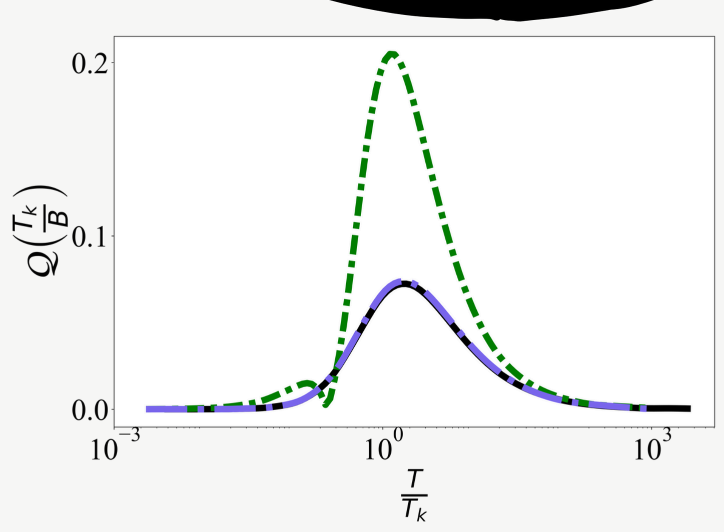


Fig. 8: Rescaled temperature sensitivity for flat band, Gaussian, and twisted bilayer graphene DOS in the universal regime B << Tk.

References

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